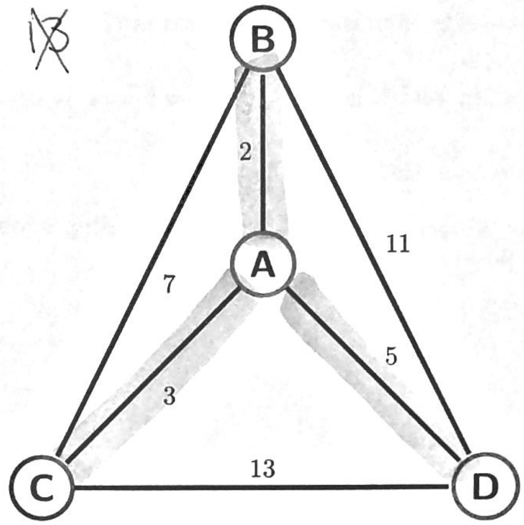
Math 1060: Worksheet June ~~9~~ 12, 2017 Name: Key

Side one of this worksheet covers material from yesterday's class (Hamiltonian circuits, Traveling Salesman Problem, and Algorithms) while the back side covers material from today's lecture (Heuristic algorithms and greedy algorithms)

1. Use Kruskal's algorithm to find a minimal spanning tree on this graph:

(2) (3) (5) ~~7~~ ~~11~~ ~~13~~

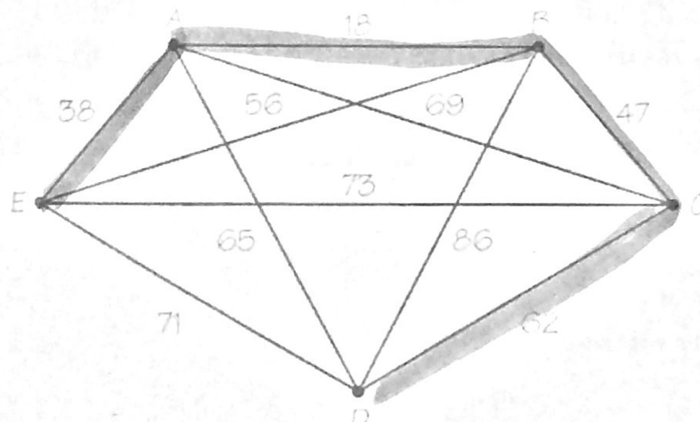
Total Cost: 10.



2. Use Kruskal's algorithm to find a minimal spanning tree in the following graph:

Sorted:

(18), (38), (47), ~~56~~, (62), ~~65~~, ~~69~~, ~~71~~, ~~73~~, ~~86~~



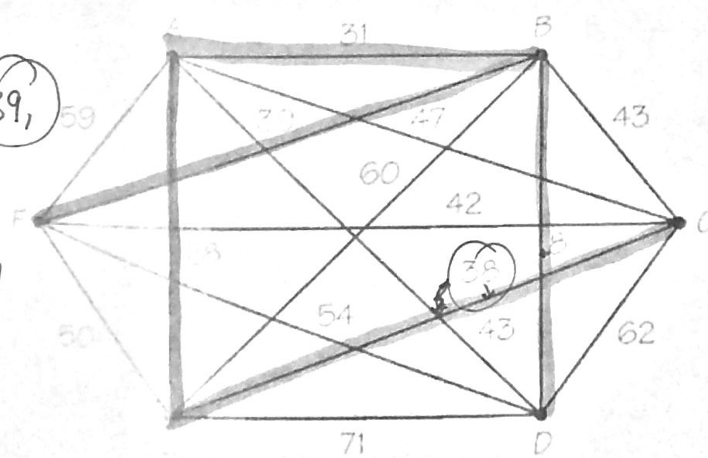
3. Use Kruskal's algorithm to find a minimal spanning tree in the following graph:

Sorted:

(8), (28), (31), (38), (39),

42, 43, 43, 47, 50,

54, 59, 60, ~~72~~ 62, 71



4. Determine whether each of the following statements is true or false for a minimum-cost spanning tree T on a weighted connected graph G.

   (a) True T contains a cheapest edge.

   (b) False T contains a most expensive edge of G.

   (c) True T contains one fewer edge than there are vertices in G.

   (d) False There is some vertex in T to which all others are joined by edges.

   (e) False There is some vertex in T that has valence 3.

5. Give an example of a real-world situation that can be modeled using a weighted graph and for which finding the minimum-cost spanning tree would be useful.

Any time you want to provide a service using pipes, wires, etc. to many houses.

6. A connected graph G has 14 vertices.

   (a) How many edges does a spanning tree of G have?

   13

   (b) How many vertices does a spanning tree of G have?

   14

   (c) What can one say about the number of edges G has?

   Since G is connected it has at least 13. (Or because T is smaller than G, G must have at least as many edges as T.)

7. Find all the spanning trees in the following graph.

Oops, there are lots.

Here's the optimal one.