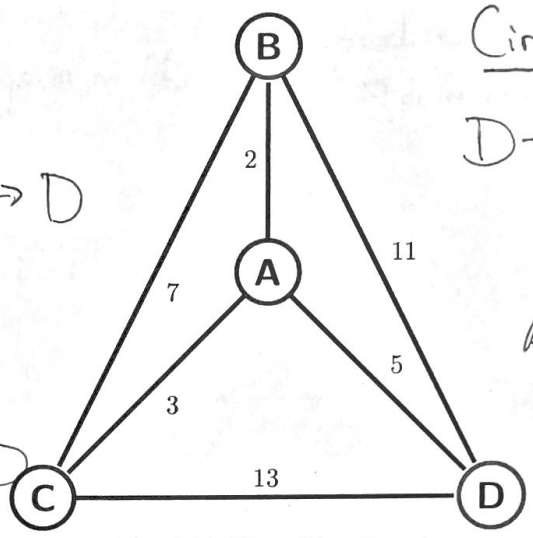
Math 1060: Worksheet June 9, 2017 Name: Key

Side one of this worksheet covers material from yesterday's class (Hamiltonian circuits, Traveling Salesman Problem, and Algorithms) while the back side covers material from today's lecture (Heuristic algorithms and greedy algorithms)

1. Solve the traveling salesman problem on the following graph by finding the total cost of all three distinct circuit.



Circuit 1

$D \xrightarrow{11} B \xrightarrow{2} A \xrightarrow{3} C \xrightarrow{13} D$

$11+2+3+13 = 29$

Circuit 2

$D \xrightarrow{5} A \xrightarrow{2} B \xrightarrow{7} C \xrightarrow{13} D$

$5+2+7+13 = 27$

Circuit 3

$D \xrightarrow{11} B \xrightarrow{7} C \xrightarrow{3} A \xrightarrow{5} D$

$11+7+3+5 = 28$

Answer:

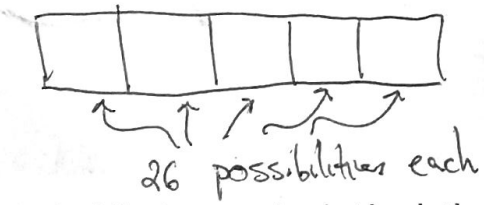
Circuit 2 is shortest.

2. (a) A lottery game requires players to select an upper case or lower case letter followed by two odd digits. How many possible entries are there?

Fund. Principle of Counting:

$52 \cdot 5 \cdot 5 = 1300$ possible entries.

(b) There are 5,757 five-letter words in the English language. What percentage of five-letter strings (i.e., five letters in a row that are not necessarily a word) are words?



26^5 possible strings.

$$\%\text{ of 5-letter strings that are words} = \frac{\#\text{ words}}{\#\text{ strings}} \cdot 100\% = \frac{5757}{26^5} \cdot 100\% = .048\%$$

3. In the following scenarios decide whether finding an Euler circuit or Hamiltonian circuit would be most helpful. Give a brief reason why.

(a) A fisherman has several traps to check each day.

HC

(b) A street sweeper needs to find a route.

EC

(c) FedEx is delivering packages to several locations in town.

HC

(d) USPS is delivering mail to houses on each street in town.

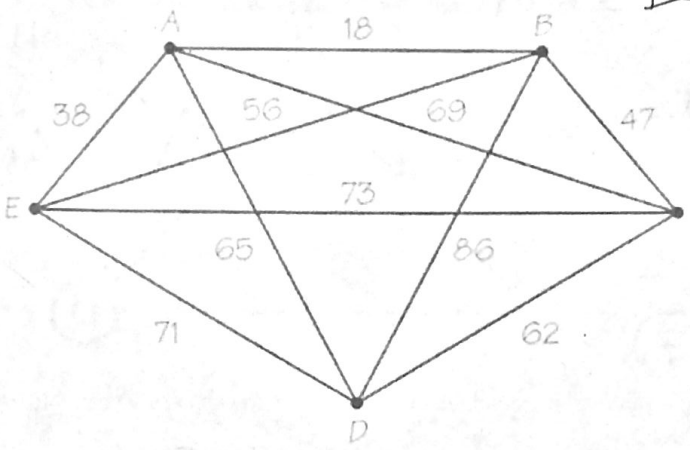
EC

4. Choose something you do every day. Write an algorithm to accomplish this task. (4-6 lines)

Make Coffee:

1. Put water in kettle
2. Put kettle on heat until 208°F
3. Grind 18g coffee beans
4. ~~Add ground beans~~ Wet filter & add to aeropress
5. Add coffee to aeropress
6. Add water.
7. Wait 90 seconds
8. Press into mug.
9. ~~Sip~~ Sip & Enjoy

5. Consider the following graph:



(a) Apply the nearest-neighbor algorithm starting at vertex A. What is the cost?

A $\xrightarrow{18}$ B $\xrightarrow{47}$ C $\xrightarrow{62}$ D $\xrightarrow{71}$ E $\xrightarrow{38}$ A

Total Cost: 18+47+62+71+38 = 236

(b) Apply the nearest-neighbor algorithm starting at vertex B. What is the cost?

B $\xrightarrow{18}$ A $\xrightarrow{38}$ E $\xrightarrow{71}$ D $\xrightarrow{62}$ C $\xrightarrow{47}$ B

Same Cost as before. (Should have checked examples better first :)

(c) Apply the sorted edges algorithm. What is the cost?

Sorted (18) (38) (47) ~~5/6~~ (62) ~~6/5~~ ~~6/7~~ (71) ~~7/3~~ ~~8/6~~

Wow this was a really bad example - oops. Cost is 236 again.

6. How many distinct Hamiltonian circuits are there in a K_6?

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ circuits

but starting position does not matter $(\div 6)$

& direction does not matter $(\div 2)$ $\rightsquigarrow \frac{720}{6 \cdot 2} = 60$ circuits