By providing my signature below I acknowledge that this is my own work, and I did not get any help from anyone else:

Name (sign): $\qquad$ Name (print): $\qquad$
Student Number: $\qquad$

| Problem <br> Number | Points <br> Possible | Points <br> Made |
| :---: | :---: | :--- |
| 1 | 8 |  |
| 2 | 6 |  |
| 3 | 16 |  |
| 4 | 30 |  |
| 5 | 20 |  |
| 6 | 20 |  |
| Total: | 100 |  |

- This test is 7 pages long. Make sure you have all 7 pages.
- If you need extra space use the last page.
- Please show your work. An unjustified answer may receive little or no credit.
- Your work must be neat. If I can't read it (or can't find it), I can't grade it.
- Please turn off your mobile phone.
- Calculators are prohibited.

1. (8 points) In the adjusted winner procedure, the Initial Winner transfers an item to the Initial Loser.
(a) (3 pts) When this transfer happens, does the total value increase, decrease, or stay the same? (Total value means the sum of Player 1's value and Player 2's value.)
(b) (5 pts) Explain your answer
2. ( 6 pts ) In the Knaster inheritance procedure, what specifically would go wrong if we sold the inherited item to the lowest bidder instead?
3. (16 points) A coffee shop has 83 employees and five locations. Below is a chart containing the average number of transactions daily at each location.

| Location | Daily Transactions |
| :---: | :---: |
| Location 1 | 1023 |
| Location 2 | 456 |
| Location 3 | 586 |
| Location 4 | 1547 |
| Location 5 | 8320 |

Consider the problem of apportioning employees to locations based on their average daily transactions.
(a) (4 pts) What are the states?
(b) (4 pts) What is the house size?
(c) (4 pts) What is the population of each state?
(d) (4 pts) What is the standard divisor?
4. (30 points) You are planning to buy 13 shirts. In order to make an informed decision, you have recorded how many times you wore each type of shirt over the past year.
(a) (12 pts) Complete this table using the Jefferson method.

| Year | Days Worn | SD $\approx 28.077$ |  | Divisor = 30 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Exact | Round | Exact | Round |
| Crew Neck | 74 | 2.636 |  | 2.467 |  |
| V Neck | 101 | 3.597 |  | 3.367 |  |
| Button Down | 164 | 5.841 |  | 5.467 |  |
| Tank Top | 26 | 0.926 |  | 0.867 |  |
| Total | 365 | 13.000 |  | N/A |  |

(b) (3 pts) You've found a divisor which gives a fair division under the Jefferson method. Is it...
(a) Less than 28.077
(b) Exactly 28.077
(c) Between 28.077 and 30
(d) Exactly 30
(e) Greater than 30
(c) (12 pts) Complete this table using the Webster method.

| Year | Days Worn | SD $\approx 28.077$ |  | Divisor = 30 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Exact | Round | Exact | Round |
| Crew Neck | 74 | 2.636 |  | 2.467 |  |
| V Neck | 101 | 3.597 |  | 3.367 |  |
| Button Down | 164 | 5.841 |  | 5.467 |  |
| Tank Top | 26 | 0.926 |  | 0.867 |  |
| Total | 365 | 13.000 |  | N/A |  |

(d) (3 pts) You've found a divisor which gives a fair division under the Webster method. Is it...
(a) Less than 28.077
(b) Exactly 28.077
(c) Between 28.077 and 30
(d) Exactly 30
(e) Greater than 30
5. (20 points) A mathematics professor is retiring and giving away her book collection. Her two PhD students decide to use the adjusted winner procedure to divide her book collection based on subject. They make the following bids on items.

| Subject | Caitlin | Bruno |
| :---: | :---: | :---: |
| Relativity | 30 | 10 |
| Abstract algebra | 15 | 45 |
| Algebraic Geometry | 20 | 15 |
| Differential Geometry | 30 | 20 |
| Algebraic topology | 5 | 10 |

(a) (5 pts) What is the initial division of items?
(b) (15 pts) Finish applying the adjusted winner procedure. What is the final allocation?
6. (20 pts) A small startup is going out of business and needs to divide its assetts equally between its three investors using the Knaster Inheritance procedure. They bid as follows.

|  | Investor 1 | Investor 2 | Investor 3 |
| :---: | :---: | :---: | :---: |
| Computers | $\$ 1500$ | $\$ 1750$ | $\$ 2250$ |
| Software | $\$ 2000$ | $\$ 1600$ | $\$ 1800$ |
| Furniture | $\$ 1000$ | $\$ 750$ | $\$ 1400$ |

Apply the Knaster inheritance procedure to obtain a fair division.

