By providing my signature below I acknowledge that this is my own work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Problem Number	Points Possible	Points Made
1	24	
2	5	
3	20	
4	10	
5	7	
6	8	
7	15	
8	6	
9	5	
Total:	100	

- This test is 8 pages long. Make sure you have all 8 pages.
- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- Please turn off your mobile phone.
- Calculators are prohibited.

1. (24 points) Consider the following four-candidate, seven-voter election.

	Number of votes					
	3	2	1	1	1	1
1^{st}	A	C	B	D	D	A
2^{nd}	B	D	C	B	C	D
3^{rd}	C	B	D	C	B	B
4^{th}	D	A	A	A	A	C

(a) (8 pts) Draw the majority rule digraph to determine a Condorcet winner, if one exists. If one does not exist, say why.

(b) (3 pts) Apply Hare method to determine a winner for this election. Show all your steps and state why this candidate is the winner.

(c) (3 pts) Apply plurality voting to determine a winner. Show your work and state why this candidate is chosen as the winner.

(d) (5 pts) Apply sequential pairwise voting with the agenda D C B A. Show all your work and state why this candidate is chosen as the winner.

(e) (5 pts) Apply the Borda count to determine a winner. Show all your work and state why this candidate is chosen as the winner.

2. (5 pts) In the following election, plurality voting is used.

Ballots

1^{st}	A	**A**	A	C	C
2^{nd}	B	**C**	C	B	A
3^{rd}	C	**B**	B	A	B

The second voter (whose ballot appears in bold) realized that by changing his preference list to C A B, he can change the outcome of the election. Is this an example of manipulation? Why or why not?

3. (20 points) Six voters, A, B, C, D, E, and F use voting system [31 : 14, 12, 9, 7, 7, 7]. For each of the following coalitions, answer the following: Is the coalition a winning coalition? Circle the critical voters. Is the coalition minimal? What is the voting weight of the coalition?

   (a) (5 pts) Coalition: B C D E

   Winning? Yes / No Minimal? Yes / No

   Circle critical voters: B C D E

   Voting weight: _____

   (b) (5 pts) Coalition: A B E

   Winning? Yes / No Minimal? Yes / No

   Circle critical voters: A B E

   Voting weight: _____

   (c) (5 pts) Coalition: A B D E

   Winning? Yes / No Minimal? Yes / No

   Circle critical voters: A B D E

   Voting weight: _____

   (d) (5 pts) Coalition: C D E F

   Winning? Yes / No Minimal? Yes / No

   Circle critical voters: C D E F

   Voting weight: _____

4. (10 points) Four voters A B C D use the voting system [14 : 6, 5, 4, 3].
   (a) (5 pts) List all winning coalitions and identify the critical voters.

   (b) (5 pts) Calculate the Banzhaf power index of each voter.

5. (7 pts) I am one of ten people who will be voting on legislation. This is my bill, so I will absolutely be voting yes. If the bill needs six votes to pass, how many ways are there for me to choose five additional voters for my coalition?

6. (8 pts) Voters A, B, C, D, E, F use the same weighted voting system as the previous question, namely [31: 14, 12, 9, 7, 7, 7]. In the following voter permutations, identify the pivotal voters.

   (a) A B C D E F

   (b) F E D C D A

   (c) A B C F E D

   (d) B F E C D A

7. (15 points) Three voters, A, B, and C, are using the weighted voting system [7: 4, 3, 3].

   (a) (3 pts) How many voter permutations exist in this voting system?

   (b) (6 pts) List all voter permutations, and identify the pivotal voter in each.

   (c) (6 pts) Calculate the Shapley-Shubik power index of each voter.

8. (6 pts) When planning a movie night, nine of the mathematics graduates students devised a weighted voting system in which the host gets a large number of votes, with the other eight attendees each receiving one vote. The host's Shapley-Shubik Power Index is 1/5. Calculate the Shapley-Shubik Power Index of each of the other attendees.

9. (5 pts) Imagine a weighted voting system with 113 voters in which *everyone* receives the same voting power. What is the Shapley-Shubik power index of each voter?

Extra space for work.