$\qquad$
Score: $\qquad$ / 168 points

1. (9 points) Give precise mathematical definitions for the following terms.
(a) Approval voting:
(b) Agenda:
(c) Plurality voting:
2. (9 points) Which term from class has the following definition?
(a) $\qquad$ A candidate who would win a one-on-one election against any of their challengers.
(b) $\qquad$ A voting system in which ballot changes preferable to one candidate (and not preferable to any other candidate) can never hurt that candidate.
(c) $\qquad$ A voting system for elections with two candidates in which the candidate with over half the total votes is the winner.
3. (6 points) Give an example of each of the following types of ballots.
(a) A preference list ballot.
(b) An approval-voting ballot.
4. (20 points) Describe each of the following properties. Why is each property desirable or undesirable? (a) (5 points) The Pareto Condition
(b) (5 points) Condorcet's Winner Criterion
(c) (5 points) Monotonicity
(d) (5 points) Independence of Irrelevant Alternatives
5. (20 points) The following example illustrates how badly the Hare system can fail to satisfy monotonicity. Consider the following sequence of preference lists:

|  | Number of voters |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Rank | 7 | 6 | 5 | 3 |
| First | A | B | C | D |
| Second | B | A | B | C |
| Third | C | C | A | B |
| Fourth | D | D | D | A |

(a) (5 points) Show that A is the unique winner if the Hare system is used.
(b) (10 points) Find the winner using the Hare system in the new election, wherein the three voters on the right all move A from last place on their preference lists to first place on their preference lists.
(c) (5 points) Explain what is surprising.
6. (8 points) In a few sentences, explain why (with an odd number of voters) Sequential Pairwise voting will always produce a unique winner.
7. (38 points) Consider the following set of preference lists and calculate the winner based using the following voting systems.

|  | Number of Voters (9) |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |  |
| First | $A$ | $A$ | $B$ | $B$ | $C$ | $C$ | $D$ |  |
| Second | $D$ | $B$ | $C$ | $C$ | $B$ | $D$ | $C$ |  |
| Third | $B$ | $C$ | $D$ | $A$ | $D$ | $B$ | $B$ |  |
| Fourth | $C$ | $D$ | $A$ | $D$ | $A$ | $A$ | $A$ |  |

(a) (10 points) Condorcet's method. (Draw the majority rule digraph, too!)
(b) (7 points) Borda count
(c) (7 points) Plurality voting
(d) (7 points) Hare system
(e) (7 points) Sequential pairwise voting with the agenda A B C D E.
8. (38 points) Consider the following set of preference lists and calculate the winner based using the following voting systems.

| Rank | Number of Voters (7) |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |  |
| First | C | E | C | D | A |  |
| Second | E | B | A | E | E |  |
| Third | D | D | D | A | C |  |
| Fourth | A | C | E | C | D |  |
| Fifth | B | A | B | B | B |  |

(a) (10 points) Condorcet's method. (Draw the majority rule digraph, too!)
(b) (7 points) Borda count
(c) (7 points) Plurality voting
(d) (7 points) Hare system
(e) (7 points) Sequential pairwise voting with the agenda E D C B A.
9. (20 points) The 10 members of a board vote by approval voting on eight candidates for new positions on their board, as is indicated in the table. (An X indicates an approval vote.)

| Candidate | Voters |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | X | X | X |  |  | $X$ | X | X |  | $x$ |
| B |  | X | $x$ | X | X | $x$ | X | X | X |  |
| C |  |  | $x$ |  |  |  |  | X |  |  |
| D | X | X | $x$ | X | X |  | X | $x$ | $x$ | $x$ |
| E | X |  | $x$ |  | X |  | X |  | X |  |
| F | X |  | X | X | X | X | X | $x$ |  | X |
| G | X | X | X | X | X |  |  | $x$ |  |  |
| H |  | X |  | $\times$ |  | x |  | x |  | x |

(a) (5 points) Which candidate is chosen if just one is to be elected?
(b) (5 points) Which candidates are chosen if the top four are to be selected?
(c) (5 points) Which candidates are elected if $80 \%$ approval is necessary and at most four are elected?
(d) (5 points) Which candidates are elected if $60 \%$ approval is necessary and at most four are elected?

