

Score: \_\_\_\_ / 154

Due June 19 at 1:00pm

1. ( \_\_\_\_ / 9 points) Give precise mathematical definitions of the following terms:

(a) Critical path:

(b) List processing algorithm:

(c) Chromatic number:

2. ( \_\_\_\_ / 9 points) What term from class has the following definition?

(a) \_\_\_\_\_: An algorithm that is fast to carry out but doesn't necessarily give an optimal solution to an optimization problem.

(b) \_\_\_\_\_: An assignment of labels to the vertices of a graph such that vertices joined by an edge get different labels.

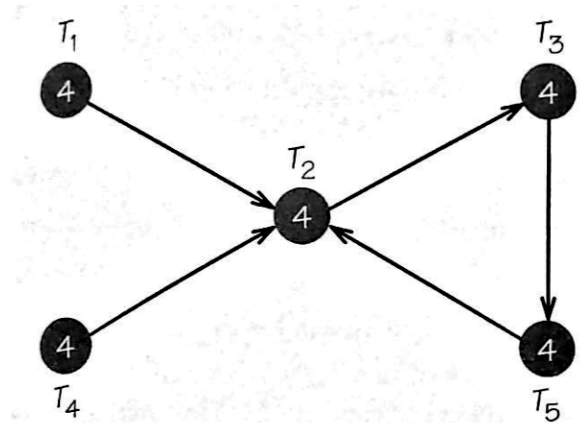
(c) \_\_\_\_\_: The study of an algorithm from the point of view of how well it performs on the hardest problems it may be used on.

3. ( \_\_\_\_ / 3 points) The following graph cannot be an order-requirement digraph because (choose one):

(a) No vertex has four edges that enter a particular vertex.

(b) All tasks require the same amount of time to complete.

(c) It has a directed circuit.

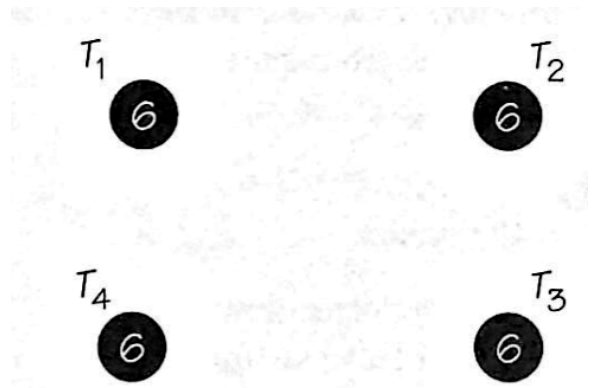


4. ( \_\_\_\_ / 3 points) Which of the following statements about the accompanying digraph is true?

(a) This graph cannot be an order-requirement digraph because it has no (directed) edges.

(b) This graph can be the order-requirement digraph of a scheduling problem.

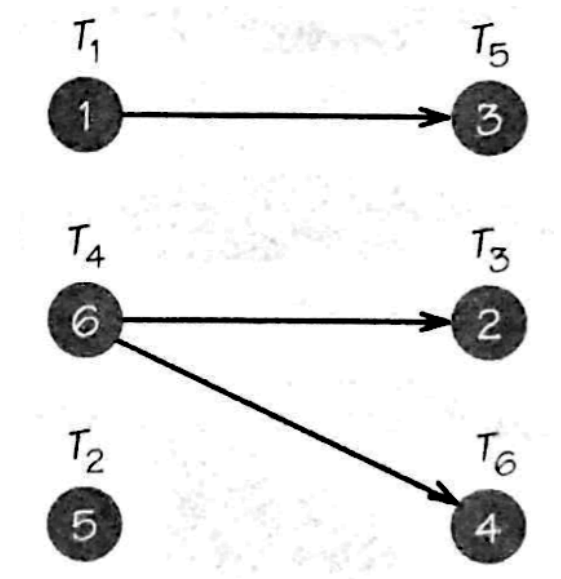
(c) This graph cannot be an order-requirement digraph because all tasks have the same time length.



5. ( \_\_\_ / 10 points) You and your two housemates are planning a party this Friday night. Eight guests are expected, and you plan to serve a homemade dinner. List the tasks involved in throwing such a party, and the types of processors used. Can any of the tasks be done simultaneously?

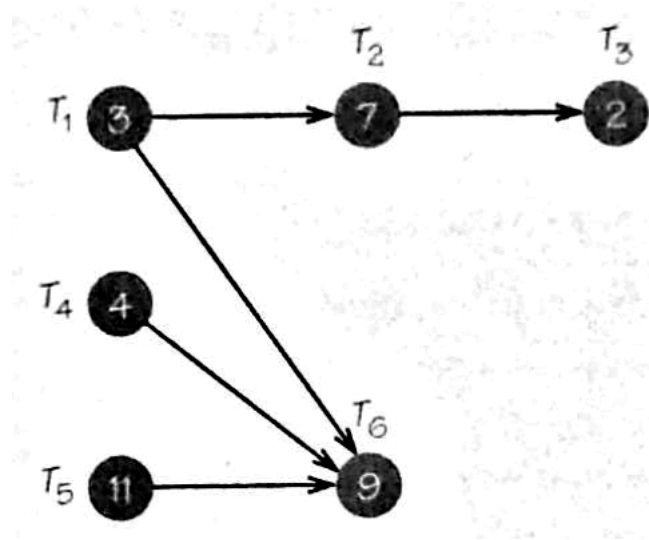
6. ( \_\_\_ / 15 points) Consider the following digraph.

- (a) ( \_\_\_ / 5 points) Schedule the jobs appearing to the right on one processor using the list processing algorithm. Use the ordering  $T_1, T_2, T_3, T_4, T_5, T_6$ .



- (b) ( \_\_\_ / 10 points) Schedule the same tasks with the same digraph and ordering using two processors instead of one.

7. ( \_\_\_ / 15 points) Answer the following questions about this order-requirement digraph.



(a) ( \_\_\_ / 4 points) What are the lengths of critical paths and which tasks are on critical paths?

(b) ( \_\_\_ / 4 points) Which task(s), taken one at a time, would not alter the length if the task were to increase by 1 time unit?

(c) ( \_\_\_ / 7 points) With two processors, can this task be scheduled to complete by time 20? If so, what list  $L$  would allow you to apply the list processing algorithm and finish by time twenty on two processors?

8. ( \_\_\_ / 25 points) At a large toy store, scooters arrive unassembled in boxes. To assemble a scooter, the following tasks must be performed:

**Task 1** Remove parts from box.

**Task 2** Attach wheels to footboard.

**Task 3** Attach vertical housing.

**Task 4** Attach handlebars to vertical housing.

**Task 5** Put on reflector tape.

**Task 6** Attach bell to handlebars.

**Task 7** Attach decals.

**Task 8** Attach kickstand.

**Task 9** Attach safety instructions to handlebars.

- (a) ( \_\_\_ / 5 points) Give reasonable time estimates for these tasks and construct a reasonable order-requirement digraph.

- (b) ( \_\_\_ / 8 points) Schedule this job on two processors (humans) using the critical path algorithm. How long does the job take? Be sure to draw the completed schedule.

- (c) ( \_\_\_ / 8 points) Schedule this job on three processors (humans) using the critical path algorithm. How long does the job take? Be sure to draw the completed schedule.

(d) ( \_\_\_ / 4 points) The cost of labor is \$10/hour. Is it more cost-effective to use two or three processors?

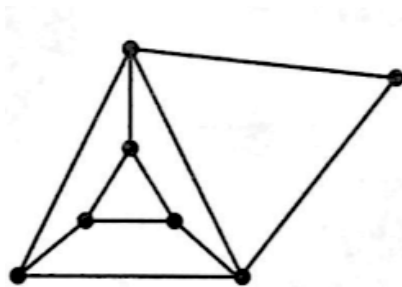
9. ( \_\_\_ / 15 points)

(a) ( \_\_\_ / 5 points) Draw a connected graph with 12 vertices and 11 edges whose vertices can be colored by two colors. What kind of graph is this?

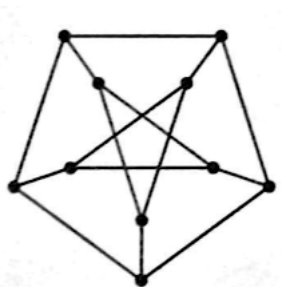
(b) ( \_\_\_ / 5 points) Can all trees be colored by two colors? Come up with a reason why, or an example of one that cannot be.

(c) ( \_\_\_ / 5 points) Add a single edge to your graph from part (a) in such a way that the resulting graph can no longer be colored with two colors.

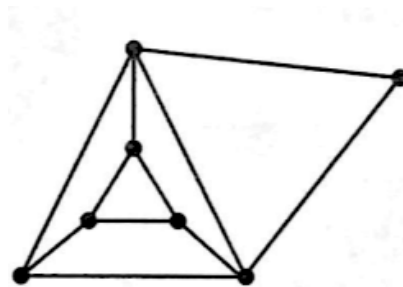
10. ( \_\_\_ / 24 points) For each of the following graphs, find its chromatic number.



(a)



(b)



(c)

11. ( \_\_\_ / 26 points) The faculty-student governing council at All State College has nine standing committees (such as Curriculum, Academic Standards, Campus Life) that are designated  $A, B, \dots, I$  for convenience. The following table shows which committees have *no member in common*.

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	<b>H</b>	<b>I</b>
<b>A</b>		X		X		X	X		X
<b>B</b>	X				X	X		X	X
<b>C</b>				X		X	X	X	X
<b>D</b>	X		X			X		X	
<b>E</b>		X					X	X	X
<b>F</b>	X	X	X	X					
<b>G</b>	X		X		X			X	
<b>H</b>		X	X	X	X		X		X
<b>I</b>	X	X	X		X			X	

- (a) ( \_\_\_ / 8 points) Draw an appropriate graph to represent the information on the table.
- (b) ( \_\_\_ / 9 points) What is the minimum number of time slots in which all the committee meetings can be scheduled?
- (c) ( \_\_\_ / 9 points) How many rooms are needed during each time slot to accommodate the committees that are scheduled to meet at that time slot?