$\qquad$
Score: $\qquad$ / 205

1. (__ / 9 points) Give precise mathematical definitions of the following terms:
(a) complete graph:
(b) tree:
(c) subgraph :
2. ( $\qquad$ / 9 points) What term from class has the following definition?
(a) $\qquad$ : A subgraph of a connected graph that is a tree and includes all vertices.
(b) $\qquad$ : A step-by-step description of how to solve a problem.
(c) $\qquad$ : A circuit that uses distinct edges of a graph to visit each vertex exactly once.
3. ( $\qquad$ / 10 points)
(a) $\qquad$ / 5 points) What is Kruskal's algorithm and what problem does it solve?
(b) ( $\qquad$ / 5 points) Write down Kruskal's algorithm in your own words.
4. ( / 5 points) You have five different coffee brewing techniques (espresso, French press, aeropress, pour over, and cold brew) and beans of four different origins (Colombia, Guatemala, Ethiopia, and Kenya). How many different cups of coffee can you brew?
5. ( $\qquad$ / 12 points) Answer these questions about the following graph:

(a) ( / 6 points) Find a Hamiltonian circuit starting at $X_{1}$.
(b) $\qquad$ / 6 points) Does there exist an Euler circuit starting at $X_{1}$ ?
6. ( _ / / 15 points)
(a) $\qquad$ / 10 points) What is the difference between a Hamiltonian circuit and an Euler circuit?
(b) (__ / 5 points) Can a circuit be both a Hamiltonian circuit and an Euler circuit? Draw an example where this happens, or explain why it can never happen.
7. ( $\qquad$ / 50 points) Answer the following questions for each graph.
(a) $($ $\qquad$ / 20 points) To the following graph, do the following:

(i) (__ / 6 points) List all three distinct Hamiltonian circuits, with their total costs.
(ii) (__ / 7 points) Apply the nearest-neighbor algorithm. Does this yield the optimal Hamiltonian circuit?
(iii) (__/ 7 points) Apply the sorted-edges algorithm. Does this yield the optimal Hamiltonian circuit? Does this agree with your answer to (ii)?
(b) (__/ 10 points) Find a minimum cost spanning tree in the following graph. What algorithm did you use? What is the cost?

(c) $($ $\qquad$ / 20 points) Answer the following questions about this graph:

(i) ( _ / / 6 points) List all three Hamiltonian circuits.
(ii) (__ / 7 points) Apply the nearest-neighbor algorithm. Is this the optimal solution?
(iii) ( _ / 7 points) Apply the sorted-edges algorithm. Is this optimal? Does it agree with your answer from (ii)?
8. ( $\qquad$ / 30 points) Consider the following graph:

(a) ( _ _ / 10 points) Apply the nearest-neighbor algorithm to the graph, starting at vertex $A$.
(b) ( _ _ / 10 points) Apply the nearest-neighbor algorithm to the graph, starting at vertex $E$.
(c) (__ / 10 points) Apply the sorted-edges algorithm to the graph.
9. ( $\qquad$ / 20 points) The following table shows distance (in miles) between four cities: Springfield (S), Urbana (U), Effington (E), and Indianapolis (I).

|  | $\boldsymbol{E}$ | $\boldsymbol{I}$ | $\boldsymbol{S}$ | $\boldsymbol{U}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{E}$ | - | 147 | 92 | 79 |
| $\boldsymbol{I}$ | 147 | - | 190 | 119 |
| $\boldsymbol{S}$ | 92 | 190 | - | 88 |
| $\boldsymbol{U}$ | 79 | 119 | 88 | - |

(a) ( $\qquad$ / 5 points) Represent this information by drawing a weighted complete graph on four vertices.
(b) ( / 10 points) Find the cost of each of the three distinct Hamiltonian circuits in the graph from part (a).
(c) (__/ 7 points) Apply the sorted-edge method to the graph of part (a).
(d) ( / 3 points) Does the sorted-edge method give the best solution?
10. (__ / 45 points) Using Google Maps or a similar service, choose five locations in Athens and answer the following questions.
(a) (__/ 7 points) List the locations, and represent the distances between them in a table similar to that of Question 9.
(b) (__ / 7 points) Represent this information by drawing a weighted complete graph on five vertices.
(c) (__ / 8 points) Apply Kruskal's algorithm to your graph. What is a real world situation where this might be useful.
(d) $\qquad$ / 8 points) Apply the nearest-neighbor algorithm to your graph.
(e) (__ / 8 points) Apply the sorted-edges algorithm to your graph.
(f) (__ / 7 points) Do the nearest-neighbor and sorted-edges algorithm agree? What is a real-world situation where this solution might be useful?

