

Math 1060: Chapters 10 & 11 Homework

Name: _____

Score: ____ / 182 points

Due when you take Test 2

1. (9 points) Give precise mathematical definitions for the following terms.
 - (a) Manipulation:
 - (b) Quota:
 - (c) May's theorem for manipulability:
2. (9 points) Which term from class has the following definition?
 - (a) _____ A voter whose vote is necessary to pass any motion.
 - (b) _____ A preference-list ballot which does not accurately represent the preferences of the voter.
 - (c) _____ A voter whose vote alone determines the outcome of an election.
3. (21 points) There are at least three voting systems for two candidates (A and B) and three voters that are nonmanipulable and that treat both candidates the same.
 - (a) (7 points) What does May's theorem tell us about such a voting system?
 - (b) (7 points) In one sentence, give an example of such a voting system.
 - (c) (7 points) Give another example.

4. (20 points) Consider the voting system that has two candidates and three voters in which the candidate with an odd number of votes is the winner. Produce two examples to show that this voting system is manipulable.

(a) (10 points) First example.

(b) (10 points) Second example.

6. (30 points) Use the following elections to show that the given voting system is manipulable.

(a) (10 points) Use the following election to show that the Hare system is manipulable.

Ballots					
1 st	A	B	B	A	A
2 nd	B	C	C	C	C
3 rd	C	A	A	B	B

(b) (10 points) Use the following election to show that the Borda count with three voters and four candidates is manipulable.

Ballots			
1 st	A	B	B
2 nd	B	A	A
3 rd	C	C	C
4 th	D	D	D

(c) (10 points) *Coomb's rule* is the voting system that operates like the Hare system, except that instead of deleting the candidate with the *fewest* first-place votes one after another, it deletes the candidates with the *most* last-place votes one after another. Use the following ballots to show that Coomb's rule is manipulable:

Ballots					
1 st	A	B	B	A	A
2 nd	B	C	C	C	C
3 rd	C	A	A	B	B

7. (20 points) Voters A, B, C, and D use the weighted voting system [50: 30, 25, 24, 21].

(a) (5 points) List all permutations in which A is pivotal.

(b) (5 points) List all permutations in which B is pivotal

(c) (10 points) Calculate the Shapley-Shubik power index of the system.

8. (10 points) D was a member of a committee until he resigned, after discovering that with the voting system in use he was a dummy. Did the Shapley Shubik power indices of the other committee members change as a result of D's departure? (Remember to justify your answer.)

9. (28 points) A, B, C, D, E, and F use the weighted voting system [15: 10, 7, 5, 3, 1, 1, 1]. Decide which of the following coalitions are winning and which are losing. Identify the critical voters in the winning coalitions.

(a) (7 points) { B, C, D, E, F }

(b) (7 points) { A, B, C }

(c) (7 points) { B, C }

(d) (7 points) { A, D, E, F }

10. (20 points) Calculate the following terms:

(a) (5 points) ${}_7C_3$

(c) (5 points) ${}_9C_5$

(b) (5 points) ${}_{15}C_2$

(d) (5 points) ${}_{100}C_{50}$ (Hint: use a computer)

11. (15 points) A committee has senior members A, B, C, and junior members D, E, and F. Each senior member has voting weight 3 while junior members have voting weight 1. The quota for passing a measure is 7. Find the minimal coalitions of the committee and determine the Banzhaf power index for members of each class.