Math 1060: Chapter 1 Homework Name: ______________________________

Score: ____ / 180 Due June 12 at 1:00pm

1. (___ / 15 points) Give a precise mathematical definition for each of the following terms:

   (a) Valence:

   (b) Graph:

   (c) Path:

2. (___ / 15 points) What term from class has the following definition?

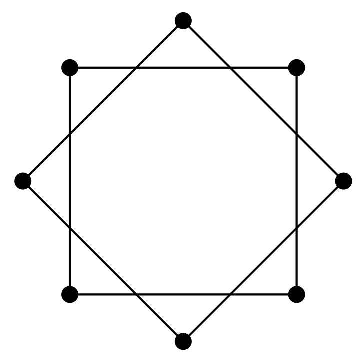
   (a) _______________: A path that starts and ends at the same vertex.

   (b) _______________: A graph in which there is a path from each vertex to any other vertex.

   (c) _______________: A mathematical statement that is proved to be true.

3. (___ / 30 points) For each of the following graphs, answer the questions: What is the number of vertices? What is the number of edges? Is the graph connected? Does each vertex have even valence? Does the graph have an Euler circuit?

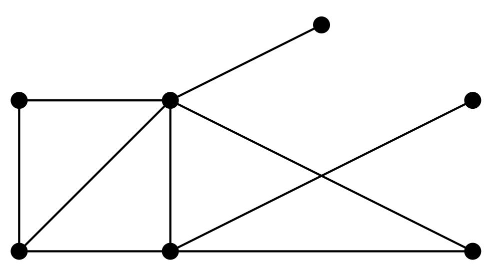
   (a) (___ / 10 points)



Vertices: _______ Edges: _______ Connected: _______

Even Valence: _______ Euler Circuit: _______

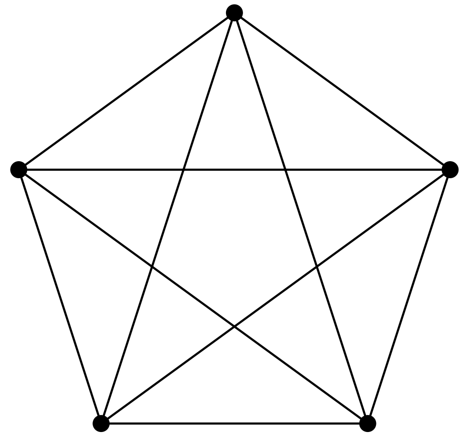
   (b) (___ / 10 points)



Vertices: _______ Edges: _______ Connected: _______

Even Valence: _______ Euler Circuit: _______

(c) (___ / 10 points)



Vertices: _______ Edges: _______ Connected: _______

Even Valence: _______ Euler Circuit: _______

4. (___ / 4 points) Answer the following multiple choice questions by circling the correct answer.

   (i) In which of the following situations would a city planner most want to find an Euler circuit on a graph?

   (a) Sweeping sidewalks of a small town.

   (b) Planning a new highway.

   (c) Planning a parade route.

   (ii) It is not possible for a graph to have five vertices of valence 3, four vertices of valence 4, and two vertices of valence 2 because:

   (a) There are no graphs with exactly 11 vertices.

   (b) A graph cannot have an even number of 4-valent vertices.

   (c) A graph cannot have an odd number of odd-valent vertices.

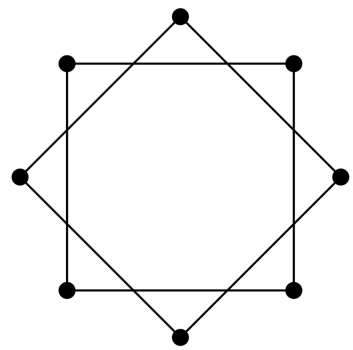
5. (___ / 16 points) Draw a graph with the given number of edges, and vertices of the given valences. If it is impossible to do so, say why.

   (a) 4 edges, 4 vertices. Valences: 1, 2, 3, and 3.

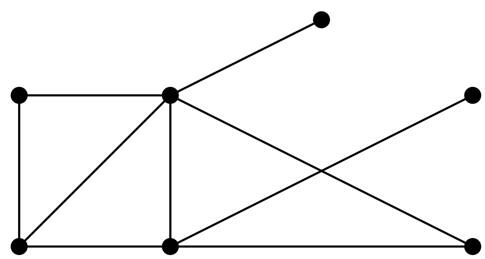
   (b) 10 edges, 10 vertices. Each vertex of valence 2.

6. (___ / 30 points) Reproduced below are the same graphs as in Question 3. If the graph has an Euler circuit, find one. Otherwise, Eulerize the graph as explained in class and then find an Euler circuit. Make sure I can tell what your answer is.

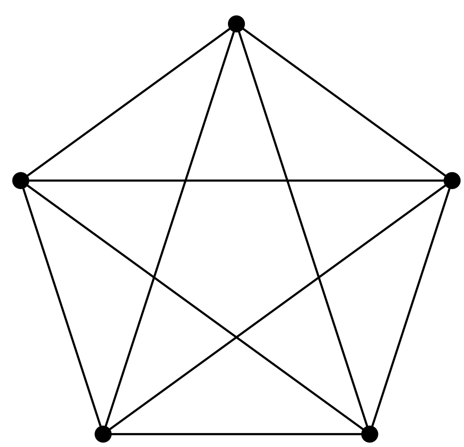
(a) (___ / 10 points)



(b) (___ / 10 points)



(c) (___ / 10 points)



7. (___ / 30 points)

(a) (___ / 10 points) Draw the complete graph on n vertices, when $n = 2, 3, 4, 5, 6, 7, 8, 9$.

(b) (___ / 5 points) For which of the above n does the complete graph on n vertices have an Euler circuit? (Circle those that do.)

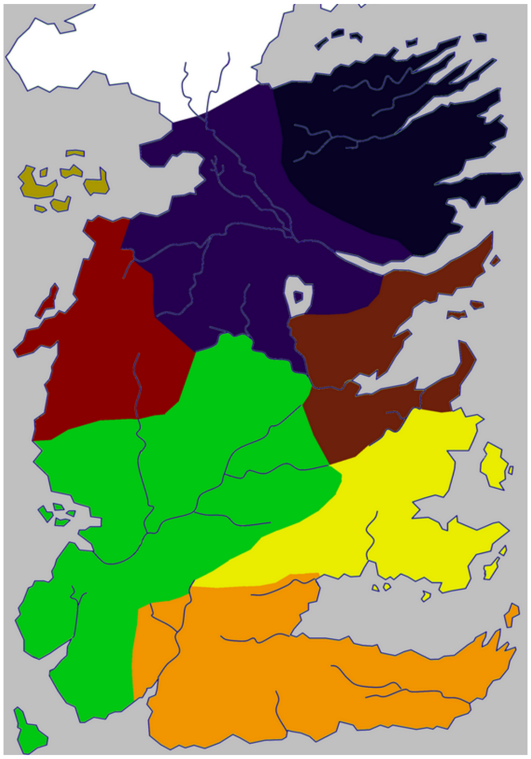
2 3 4 5 6 7 8 9

(c) (___ / 7 points) Use your answer to (b) to guess for which n (*any* integer) the complete graph on n vertices has an Euler circuit.

(d) (___ / 8 points) Explain why your guess should be true, using what we've learned in class.

8. (___ / 15 points)

(a) (___ / 10 points) Here is a map of the seven kingdoms of Westeros. Draw a graph whose vertices are the seven kingdoms, with edges connecting kingdoms which share a border. (The white region at the top is “beyond the wall” and is not a kingdom. The Iron Islands near the top left also are not a kingdom.)



(b) (___ / 5 points) Consider the graph from part (a). What does the valence of a vertex represent?

9. (___ / 25 points)

(a) (___ / 10 points) Pick a neighborhood in Athens and draw a graph whose vertices are intersections and whose edges are streets.

(b) (___ / 5 points) Can a graph such as that in part (a) be disconnected? Why or why not?

(c) (___ / 10 points) Solve the chinese postman problem for the graph you made in part (a).