Department of Mathematics
University of Georgia

Final Exam

Math 1060
July 28, 2017

By providing my signature below I acknowledge that this is my own work, and I did not get any help from anyone else:

Name (sign): ____________________

Name (print): ____________________

Student Number: ____________________

Answer Key

Problem Number	Points Possible	Points Made
1	10	
2	8	
3	16	
4	14	
5	14	
6	10	
7	10	
8	15	
9	17	
10	24	
11	8	
12	8	
13	30	
14	5	
15	5	
16	4	
17	10	
18	6	
Total:	214	

- This test is 18 pages long. Make sure you have all 18 pages.
- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- Please turn off your mobile phone.
- Calculators are prohibited.

1. (10 points) This problem will lead you to count the number of edges in a complete graph on n vertices.

   (a) (3 pts) What is the valence of any one vertex in a complete graph which has n vertices?

   Complete Graph: Each vertex connected to all others.

   $\boxed{n-1 \text{ other vertices}}$

   (b) (3 pts) What is the sum of all valences?

   $$\underbrace{(n-1) + (n-1) + \ldots + (n-1)}_{n \text{ times (once for each vertex.)}} = \boxed{n \cdot (n-1).}$$

   (c) (4 pts) Use your answer to part (b), and an equation from class to determine the number of edges in this complete graph on n vertices. Which equation from class did you use?

   $$\text{Total Valence} = 2 \cdot \text{Edges}$$

   $$n \cdot (n-1) = 2 \cdot \text{Edges}$$

   $$\boxed{\frac{n \cdot (n-1)}{2}} = \text{Edges}$$

2. (8 points) Answer the following as True or False by circling one option. (You do not need to justify your answer.)

   (a) (4 pts) Every complete graph with at least 3 vertices has a Hamiltonian circuit.

   [✓ True] False

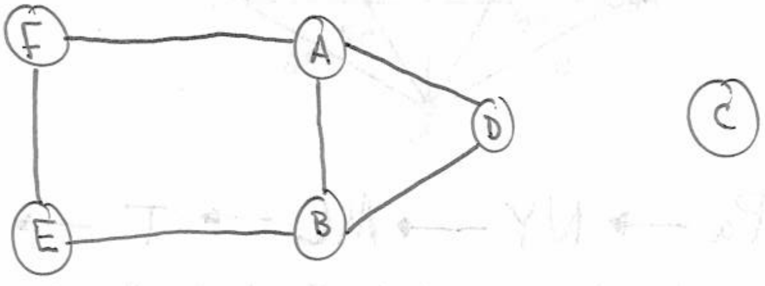
   (b) (4 pts) Every complete graph with at least 3 vertices has an Euler circuit.

   True [✓ False]

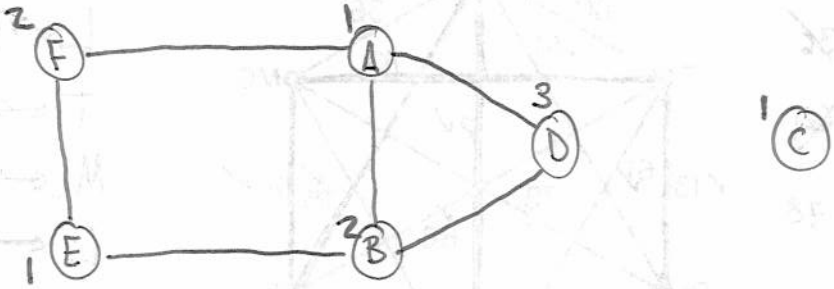
3. (16 points) A company manufactures 6 chemicals. Certain pairs of chemicals will cause explosions if brought into contact with each other, as is shown in the following table. (An x indicated that the chemicals have an explosive interaction.)

	A	B	C	D	E	F
A		x		x		x
B	x			x	x	
C						
D	x	x				
E		x				x
F	x				x	

(a) (6 pts) Draw these relations as a graph, where vertices correspond to chemicals and edges connect chemicals that cause explosions if brought into contact. (Label your vertices!)



(b) (6 pts) The company wants to store these chemicals in warehouses so that no reactant pairs will be stored in the same warehouse. Find a 3-coloring of your graph from (a) to determine a way to store these chemicals in three warehouses.

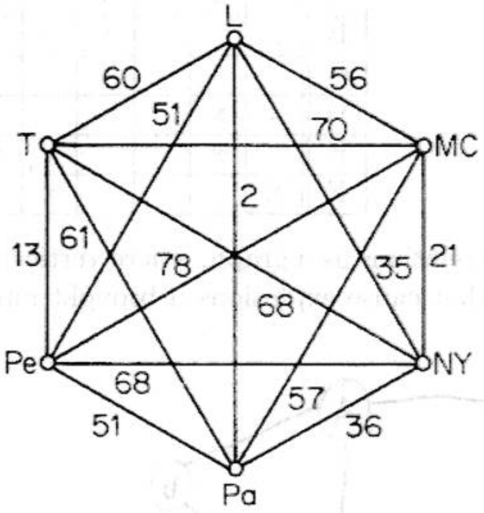


(c) (4 pts) What is represented by the chromatic number of this graph?

Smallest number of warehouses necessary to store chemicals w/ no reactive chemicals sharing a warehouse.

4. (14 points) In this question, the same graph is reproduced multiple times.

(a) (7 pts) Use the nearest-neighbor algorithm to find a Hamiltonian circuit in the following graph, starting at vertex L. Write your answer as an ordered list of vertices.

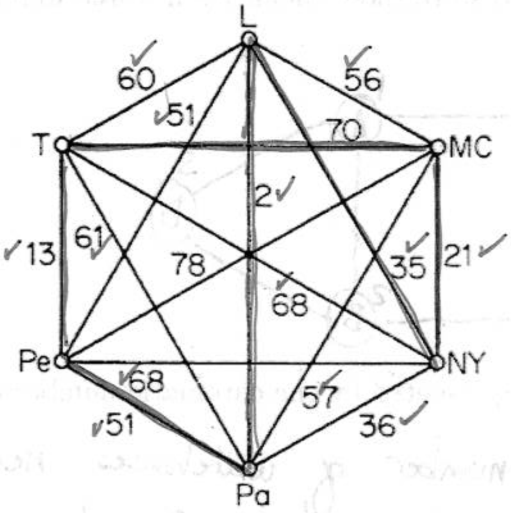


L → Pa → NY → MC → T → Pe → L

(b) (7 pts) Use the sorted-edges algorithm to find a Hamiltonian Circuit in the following graph. Write your answer as a list of edges in your circuit.

List of edges:

② ⑬ ㉑ ㉟ ~~36~~
(51) ~~51~~ ~~56~~ ~~57~~ ~~60~~
~~61~~ ~~68~~ ~~68~~ (70) 78



Edges in Circuit

L ↔ Pa
T ↔ Pe
Mc ↔ NY
L ↔ NY
Pe ↔ Pa
T ↔ Mc

5. (14 points) Euler's theorem states that a graph has an Euler circuit if and only if two conditions hold.

   (a) (6 pts) What are the two conditions?

Graph is connected

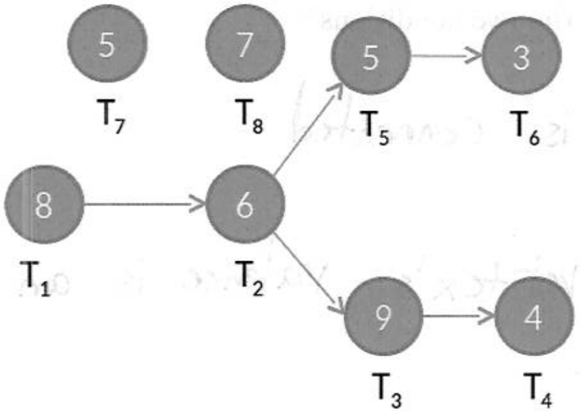
Every vertex's valence is an even number

   (b) (8 pts) Explain why each condition is necessary. (E.g., "`[Condition 1]` is necessary because if a graph didn't satisfy `[Condition 1]` it would not have an Euler circuit for `[reason]`". Same for `Condition 2`.)

If graph wasn't connected, you definitely couldn't make a circuit that visits each edge since edges might lie on different connected pieces.

Your circuit has to leave each vertex that it enters, using different edges. So for each enter/exit we add 2 valence.

6. (10 points) Consider the following order requirement digraph.



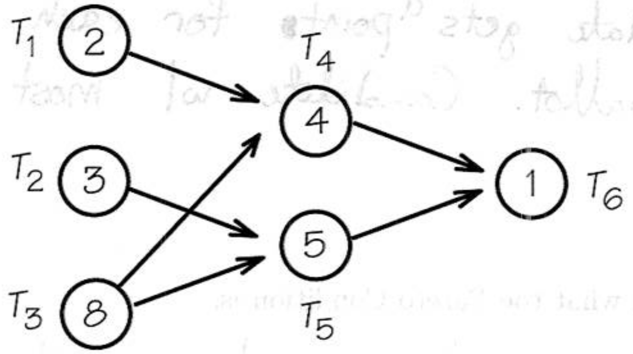
(a) (4 pts) What is the critical path algorithm used to generate?

Priority list.

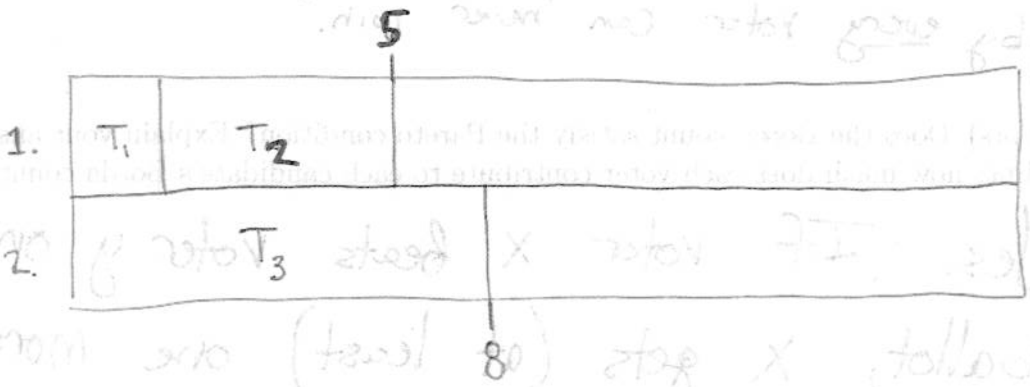
(b) (6 pts) Apply the critical path algorithm.

T_1 T_2 T_3 T_5 T_8 T_7 T_4 T_6

7. (10 points) I'm applying the list-processing algorithm to a job with the following order requirement digraph:



on two processors with the priority list $T_1, T_3, T_4, T_2, T_6, T_5$. Here is my progress so far.



(a) (5 pts) What task should be scheduled on processor 1 at time 6?

No task should be scheduled.

(b) (5 pts) What task is scheduled on processor 2 at time 8?

T_4 or T_5.

8. (15 points)

(a) (5 pts) Explain what the Borda Count Voting System is.

Each candidate gets a point for each candidate they beat on each ballot. Candidate w/ most points wins.

(b) (5 pts) Explain what the Pareto Condition is.

It is a property a voting system could have or not have. The property is: "A candidate less preferred (than another candidate) by every voter can never win."

(c) (5 pts) Does the Borda count satisfy the Pareto condition? Explain your answer. (Hint: how much does each voter contribute to each candidate's Borda count?)

Yes. If voter X beats voter y on a ballot, X gets (at least) one more point. As a whole, this means X's total points are at least Y's total points plus the number of voters.

9. (17 points) Consider the following election with candidates A, B, and C:

	Number of Voters			
	5	4	3	1
1^{st}	A	C	B	B
2^{nd}	B	B	C	A
3^{rd}	C	A	A	C

(a) (3 pts) Who is the Hare winner in this election?

A.

(b) (4 pts) If the voter with the right-most ballot switched their ballot to A B C, who would be the Hare winner?

C.

(c) (4 pts) Is this an example of ballot manipulation? Why?

No. C is less preferred by voter whose ballot changed.

(d) (6 pts) Is this an example that the Hare Voting System doesn't satisfy a property from class? What property? How does this example show Hare Voting doesn't satisfy the property?

Yes, this is a failure of monotonicity.

10. (24 points) Four voters, A B C D use the weighted voting system [9 : 5, 4, 2, 2].

(a) (6 pts) Identify the pivotal voter in each of the voting permutations, listed below.

A[✓ B]CD	A[✓ B]DC	AC[✓ B]D	AC[✓ D]B	AD[✓ B]C	AD[✓ C]B
B[✓ A]CD	B[✓ A]DC	BC[✓ A]D	BCD[✓ A]	BD[✓ A]C	BDC[✓ A]
CA[✓ B]D	CA[✓ D]B	CB[✓ A]D	CBD[✓ A]	CD[✓ A]B	CDB[✓ A]
DA[✓ B]C	DA[✓ C]B	DB[✓ A]C	DBC[✓ A]	DC[✓ A]B	DCB[✓ A]

A: 14 B: 6 C: 2 D: 2

(b) (6 pts) List all of the winning coalitions, and identify the critical voters in each.

4 voter winning Coalitions	3 voter winning Coalitions	2 voter winning Coalitions
$\underline{A}$ B C D 13	$\underline{A}\ \underline{C}\ \underline{D}$ 9	$\underline{A}\ \underline{B}$ 9
	$\underline{A}\ \underline{B}$ D 11	
	$\underline{A}\ \underline{B}$ C 11	

(c) (6 pts) Calculate the Shapley-Shubik Power Index of each voter.

A : $\frac{14}{24}$

# of permutations A is pivotal

Total permutations

B: $^{6}/_{24}$

C : $^{2}/_{24}$

D: $^{2}/_{24}$

(d) (6 pts) Calculate the Banzhaf Power Index of each voter.

A: 5

B: 3

C: 1

D: 1

11. (8 points) Consider the weighted voting system [10: 5, 2, 2, 2, 2].

    (a) (3 pts) A coalition cannot pass the vote unless the weight-five voter is in the coalition. What term from class describes this voter?

    This voter has veto-power.

    (b) (5 pts) Calculate the Shapley-Shubik Power index of each voter. Show your work.

Possible permutations of all weight 2 voters are the same:

(Pivotal voter circled)

5 2 2 ② 2

2 5 2 ② 2

2 2 5 ② 2

2 2 2 ⑤ 2

2 2 2 2 ⑤

weight 5's BPI is $\boxed{2/5}$

weight 2's BPI are all the same.

Total BPI is 1, so weight 2s sum to 3/5.

So each weight 2 voter has BPI $3/5 \div 4$, i.e.

$0.6/4 = 0.15$.

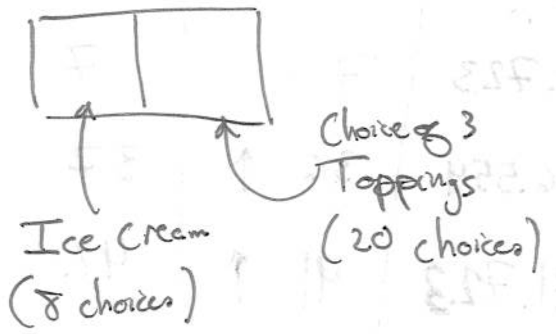
12. (8 points)

(a) (4 pts) An ice cream parlor has 6 different ice cream toppings. A sundae is vanilla ice cream with three toppings. How many ways are there to build a sundae?

$${}_6C_3 = \frac{6!}{3!\cdot(6-3)!} = \frac{6\cdot5\cdot4}{3\cdot2\cdot1} = 20$$

(b) (4 pts) In order to appeal to a broader audience, management has decided you can use any of the 8 ice cream flavors as the base of your sundae. How many possible sundaes are there now?
(Hint: Use your answer from part 1.)

Make 2 choices for a sundae: Ice cream & toppings.



so $8\cdot 20 = \boxed{160}$ diff. sundaes.

13. (30 points) A chain of instrument stores wants to restock ukuleles to their six locations, based on number of ukuleles sold in the past year. The locations have sold 84, 16, 47, 99, 113, and 31 ukuleles. Corporate is looking to restock a gross (i.e., 144) of ukuleles to their locations.

(a) (6 pts) Phrase this as an apportionment problem. What are the states, their populations, and the house size?

States : Instruments stores

Pops : # of ukes sold.

House Size: 144

(b) (3 pts) What is the standard divisor?

$$\frac{84+16+47+99+113+31}{144} \approx 2.7083$$

(c) (5 pts) Use the Hamilton method to apportion ukuleles to the locations. Make sure to draw your table and compute quotas.

	Pop	Quota	Round	Final
Loc 1	84	31.015	31	31
Loc 2	16	5.908	5 ↑	6
Loc 3	47	17.354	17	17
Loc 4	99	36.554	36 ↑	37
Loc 5	113	41.723	41 ↑	42
Loc 6	31	11.446	11	11
			141	144

(d) (8 pts) Attempt to apportion the ukuleles using the standard divisor and the Jefferson method. Choose another divisor (cleverly) and attempt to apportion using that divisor. If you had to guess what a correct divisor would be, what would you guess?

	Pop	$D_iv = SD \approx 2.7083$		Divisor $= 2.5$	
Loc 1	84	31.015	31	33.6	33
Loc 2	16	5.908	5	6.4	6
Loc 3	47	17.354	17	18.8	18
Loc 4	99	36.554	36	39.6	39
Loc 5	113	41.723	41	45.2	45
Loc 6	31	11.446	11	12.4	12
			141		133

Would guess 2.6

(e) (8 pts) Attempt to apportion the ukuleles using the standard divisor and the Hills-Huntington method. Choose another divisor (cleverly) and attempt to apportion using that divisor. If you had to guess what a correct divisor would be, what would you guess?

	Pop	Div = SD		~~Div = 2.68~~
Loc 1	84	31.015	31	~~31.343~~
Loc 2	16	5.908	6	~~5.97~~
Loc 3	47	17.354	17	~~17.537~~
Loc 4	99	36.554	37	~~36.940~~
Loc 5	113	41.723	42	~~42~~
Loc 6	31	11.446	11	
			144	

SD was right!

14. (5 pts) When applying the Adjusted Winner Procedure, we always end up with a total value of more than 100 points. How is this possible? (Answer in one or two sentences.)

Items are initially given to person who values them the most. Transferred items lose least possible.

15. (5 pts) UGA has a policy that any class which has too few enrolled students cannot run. Given this, when apportioning teachers to classes based on enrollment, which rounding technique would you use? (Hills-Huntington, Webster, or Jefferson.) Why?

Webster or Jefferson. These allow low values to be rounded down to 0. HH does not do this.

16. (4 pts) Pretend we have a binary linear code of weight four. If we add extra codewords to our binary linear code, what could the weight possibly be? (Circle all that apply.)

0 [✓ 1] [✓ 2] [✓ 3] 4 5 6+

17. (10 points) There are eight possible length-three binary strings. These are 000, 001, 010, 011, 100, 101, 110, and 111. Encode these by adding two parity-check digits, one corresponding to digits 1 and 2, and one corresponding to digits 3 and 4 (notice that 4 is a new digit – this is fine!).

(a) (4 pts) Write down all eight encoded words.

0 0 0 0 0

0 0 1 0 1

0 1 0 1 1

0 1 1 1 0

1 0 0 1 1

1 0 1 1 0

1 1 0 0 0

1 1 1 0 1

(b) (3 pts) What is the weight of this code?

Weight is least # of nonzero entries. 00101. is least, w/ 2 nonzero entries. $\boxed{\text{Weight} = 2}$

(c) (3 pts) We are sending these codewords across a noisy channel. How many errors can this code detect?

It can detect one error.

18. (6 points)

(a) (3 pts) Write down something interesting or fun you're doing once this class finishes

Road trips!

(b) (3 pts) Pat yourself on the back, double check your work, congratulate yourself, and enjoy what's left of your summer!

___ / 6 points